Calculus!
Topic 7

7.3

- Gradients of curves for given values of x

Calculus 7.3:

- Gradients of curves

Function	$f(x)$	$f'(x)$
a constant	a	0
x^n	x^n	nx^{n-1}
a constant multiple of x^n	ax^n	anx^{n-1}
multiple terms	$u(x) + v(x)$	$u'(x) + v'(x)$

Calculus 7.3:

- Gradients of curves

2 Suppose $f(x) = 4x^3 - x$. Find:

a $f'(x)$ **b** $f'(2)$ **c** $f'(0)$

3 Suppose $g(x) = \frac{x^2 + 1}{x}$. Find:

a $g'(x)$ **b** $g'(3)$ **c** $g'(-2)$

Calculus 7.3:

- Gradients of curves

5 Consider the function $f(x) = (3x + 1)^2$.

a Expand the brackets of $(3x + 1)^2$.

b Hence find $f'(x)$.

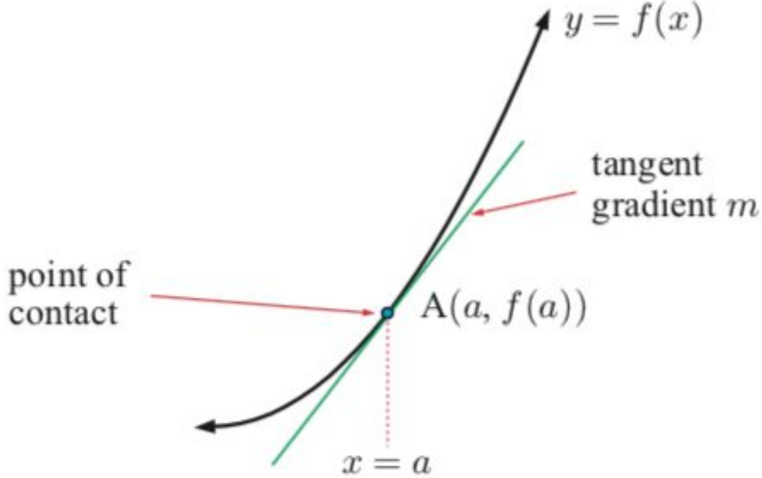
c Hence find the gradient of the tangent to $y = f(x)$ at the point where $x = -2$.

Check your answer to **c** using technology.

Calculus 7.3:

- Gradients of curves

E EQUATIONS OF TANGENTS



Consider a curve $y = f(x)$.

If the point A has x-coordinate a, then its y-coordinate is $f(a)$, and the gradient of the tangent at A is $f'(a)$.

The equation of the tangent is

$$\frac{y - f(a)}{x - a} = f'(a) \quad \{\text{equating gradients}\}$$

or $y - f(a) = f'(a)(x - a)$.

Calculus 7.3:

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Slope Intercept: $y=mx+c$

y = y coordinate

x = x coordinate

m = slope of the line

c = where the line intercepts the y axis

Calculus 7.3:

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EXERCISE 20E

1 Find the equation of the tangent to:

a $y = x^2$ at $x = 4$

b $y = x^3$ at $x = -2$

Calculus 7.3:

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EXERCISE 20E

1 Find the equation of the tangent to:

a $y = x^2$ at $x = 4$

b $y = x^3$ at $x = -2$

c $y = 3x^{-1}$ at $x = -1$

d $y = \dfrac{4}{x^3}$ at $x = 2$

e $y = x^2 + 5x - 4$ at $x = 1$

f $y = 2x^2 + 5x + 3$ at $x = -2$

g $y = x^3 + 2x$ at $x = 0$

h $y = x^2 + x^{-1}$ at $x = 0$

i $y = x + 2x^{-1}$ at $x = 2$

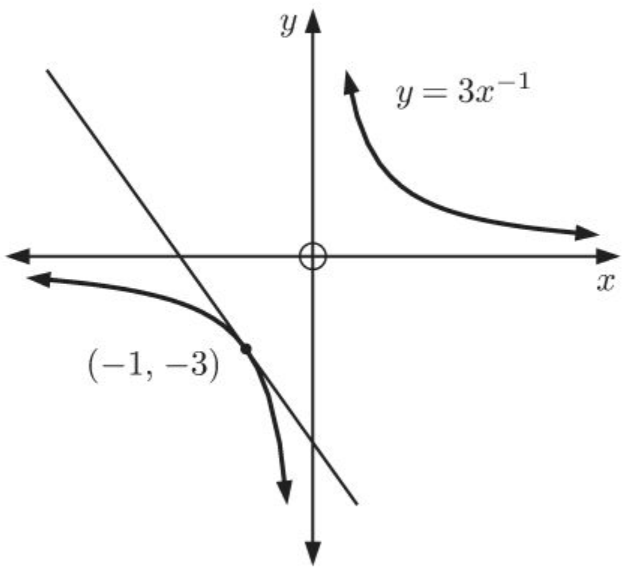
j $y = \dfrac{x^2 + 4}{x}$ at $x = -1$

Check your answers using technology.

Calculus 7.3:

- Gradients of curves

c



When $x = -1$, $y = 3(-1)^{-1} = -3$, so the point of contact is $(-1, -3)$.

Now $\dfrac{dy}{dx} = -3x^{-2} = -\dfrac{3}{x^2}$,

so when $x = -1$, $\dfrac{dy}{dx} = -\dfrac{3}{(-1)^2}$

$= -3$

$\therefore$ the tangent has equation $\dfrac{y - (-3)}{x - (-1)} = -3$

which is $y + 3 = -3x - 3$

or $y = -3x - 6$

Calculus 7.3:

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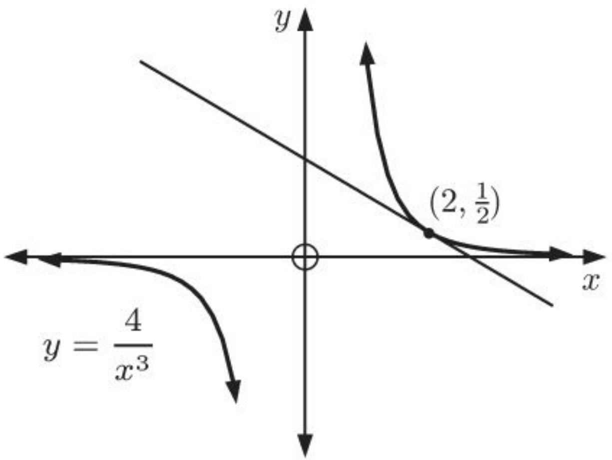
d



When $x = 2$, $y = \dfrac{4}{2^3} = \frac{1}{2}$, so the point of contact is $(2, \frac{1}{2})$.

Now $y = \dfrac{4}{x^3} = 4x^{-3}$,

$$\therefore \quad \frac{dy}{dx} = -12x^{-4} = -\frac{12}{x^4}$$

So when $x = 2$,
$$\frac{dy}{dx} = -\frac{12}{2^4} = -\tfrac{12}{16} = -\tfrac{3}{4}$$

$\therefore$ the tangent has equation $\dfrac{y - \frac{1}{2}}{x - 2} = -\frac{3}{4}$

which is $y - \frac{1}{2} = -\frac{3}{4}x + \frac{3}{2}$

or $y = -\frac{3}{4}x + 2$

Calculus 7.3:

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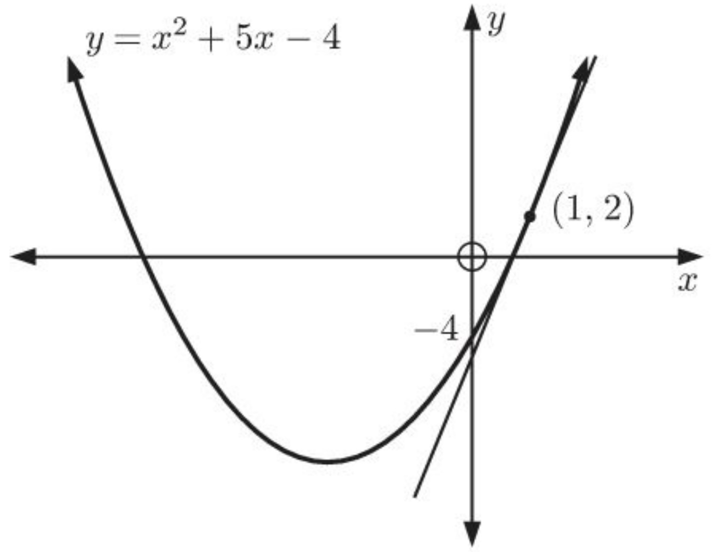
e



When $x = 1$, $y = 1^2 + 5(1) - 4 = 2$, so the point of contact is $(1, 2)$.

$$\text{Now} \quad \frac{dy}{dx} = 2x + 5,$$

$$\text{so when } x = 1, \quad \frac{dy}{dx} = 2(1) + 5 = 7$$

$$\therefore \quad \text{the tangent has equation} \quad \frac{y-2}{x-1} = 7$$

$$\text{which is} \quad y - 2 = 7x - 7$$

$$\text{or} \quad y = 7x - 5$$

Calculus 7.3:

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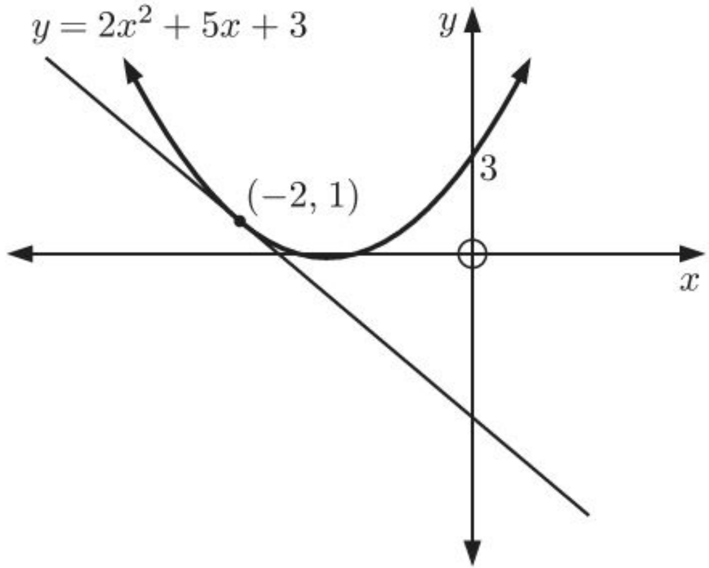
f



When $x = -2$, $y = 2(-2)^2 + 5(-2) + 3 = 1$, so the point of contact is $(-2, 1)$.

Now $\frac{dy}{dx} = 4x + 5$,

so when $x = -2$, $\frac{dy}{dx} = 4(-2) + 5$
$= -3$

$\therefore$ the tangent has equation $\frac{y - 1}{x - (-2)} = -3$

which is $y - 1 = -3(x + 2)$
$= -3x - 6$

or $y = -3x - 5$

Homework!

These are from the Oxford textbook (pink one!)

Work through as many as it takes to feel confident!

Remember they get harder as you go!

Exercise 6F

1 Find the equation of the tangent to the given curve at the stated point, P. Give your answers in the form $y = mx + c$.

a $y = x^2$; P(3, 9)
b $y = 2x^3$; P(1, 2)
c $y = 6x - x^2$; P(2, 8)
d $y = 3x^2 - 10$; P(1, –7)
e $y = 2x^2 - 5x + 4$; P(3, 7)
f $y = 10x - x^3 + 5$; P(2, 17)
g $y = 11 - 2x^2$; P(3, –7)
h $y = 5 - x^2 + 6x$; P(2, 13)
i $y = 4x^2 - x^3$; P(4, 0)
j $y = 5x - 3x^2$; P(–1, –8)
k $y = 6x^2 - 2x^3$; P(2, 8)
l $y = 60x - 5x^2 + 7$; P(2, 107)
m $y = \frac{1}{2}x^4 - 7$; P(4,121)
n $y = 17 - 3x + 5x^2$; P(0, 17)
o $y = 2x(5 - x)$; P(0, 0)
p $y = \frac{1}{4}x^3 - 4x$; P(2,–6)
q $y = \frac{3}{4}x^2 + 3$; P(–2,6)
r $y = \frac{2}{3}x^3 + \frac{1}{3}$; $P\left(-1, -\frac{1}{3}\right)$
s $y = \frac{1}{4}x^3 - 7x^2 + 5$; P(–2,–25)

2 Find the equation of the tangent to the given curve at the stated point. Give your answers in the form $ax + by + c = 0$

a $y = \frac{12}{x^2}$; (2, 3)
b $y = 5 + \frac{6}{x^3}$; (1, 11)